Advances in QCD sum-rule calculations

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Abstract. We review the recent progress in the applications of QCD sum rules to hadron properties with the emphasis on the following selected problems: (i) development of new algorithms for the extraction of ground-state parameters from two-point correlators; (ii) form factors at large momentum transfers from three-point vacuum correlation functions; (iii) properties of exotic tetraquark hadrons from correlation functions of four-quark currents.

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INTRODUCTION

The method of sum rules is 35 years old. In spite of this respectable age, the method is being permanently enriched by new ideas and new calculations and remains one of the widely used and competitive tools both for the determinations of the fundamental QCD parameters (e.g., quark masses and α_s) and for the calculation of hadron properties. In this talk we review the recent progress in the applications of QCD sum rules to hadron properties with the emphasis on the selected topics: (i) sum rules for two-point vacuum correlation functions and leptonic decay constants of heavy mesons; (ii) sum rules for three-point vacuum correlation functions, form factors and three-meson couplings; (iii) sum rules for exotic tetraquark states.

QCD sum rules [1] (see also [2, 3] for further references) is one of the main *analytic* methods for the study of hadron properties from the field-theoretic Green functions (correlators) in full QCD. The correlators are calculated by means of the Wilsonian operator product expansion (OPE) which provides the rigorous framework for the separation of long and short distances, in QCD being dominated by nonperturbative and perturbative physics, respectively [4]. The OPE clearly identifies, e.g., the origin of chiral symmetry breaking and the emergence of hadron masses, leads to factorization of complicated amplitudes of hadron interactions at large momentum transfers.

• QCD sum rules provide hadron amplitudes which satisfy all rigorous properties imposed by perturbative QCD and, at the same time, contain nonperturbative contributions determined in a unique way. As an OPE-based method, QCD sum rules are formulated in the Euclidean region. However, by combining OPE with the knowledge of the analytic structure of the Green functions and resummation schemes, the analytic continuation to the Minkowski space may be performed. In this respect QCD sum rules may have a broader range of applicability than lattice QCD. Last but not least, as an analytic method, QCD sum rules provide physics insights in the hadron structure, which are not easy to get from the numerical results of lattice QCD.

• The method of QCD sum rules favourably compares with other analytic methods, such as effective theories or functional methods: the method of sum rules is based on the Wilsonian OPE in full QCD and therefore involves no other implicit assumptions often present in other analytic method.

a. OPE and the sum rule for the correlator

The basic object in the method of QCD sum rules – as well as in lattice QCD – is the vacuum-to-vacuum correlator, i.e., the vacuum average of the T-product of quark and gluon currents. In lattice QCD, one finds this correlator numerically at large values of the Euclidean time τ. In the method of QCD sum rules, one calculates the correlator analytically as the Taylor expansion in τ. Technically, one considers a so-called Borelized correlator, i.e. applies the Borel transform to the Feynman diagrams, written as spectral representations in the energy variables. The inverse Borel mass parameter is related to τ. The OPE provides the analytic double expansion of this correlator in form of a perturbatively calculable power series in the strong coupling constant α_s and in powers of τ; the "power corrections" — terms involving powers of τ — are given via *condensates*, expectation values of gauge-invariant operators over the physical vacuum in QCD; these condensates describe in an unambiguous way nonperturbative QCD contributions.

Alternatively, one may derive a representation for the Borelized correlator in terms of the intermediate hadron states.

The two representations for the Borelized correlator — by OPE and by sum over hadron states — constitute the two sides of the QCD sum rule.

b. Isolating the ground-state contribution from the Borelized correlator

At large τ, the ground-state dominates the correlator which thus fully determines the ground-state parameters. In the region of small and intermediate τ, where the truncated OPE gives a good description of the correlator, excited hadronic states give sizeable contributions. In order to get rid of the excited states and to isolate the ground-state contribution from the correlator, one invokes the idea of quark–hadron duality [5–7]: the excited states are dual to high-energy parts of Feynman diagrams of perturbative QCD. The ground-state contribution is then equal to the "*dual correlator*" – the correlator in which the spectral integrals for perturbation theory diagrams are cut at a certain *effective continuum threshold* $s_{\rm eff}$, or simply "effective threshold". The effective continuum threshold differs from the physical continuum threshold determined by masses of low-lying hadrons. Obviously, apart from a truncated OPE for a correlator, the effective continuum threshold is a crucial ingredient of every sum-rule extraction of ground-state parameters; this quantity governs the accuracy of the quark–hadron duality and determines to large extent the numerical value of the extracted parameters of the bound state. The truncated OPE itself cannot provide precise values of the ground-state parameters. Therefore, the method of QCD sum rules provides hadron parameters with some uncertainty which is referred to as systematic uncertainty [8].

Understanding the properties of the effective continuum threshold and finding a criterion for fixing this quantity is the key to obtaining reliable hadron parameters from sum rules.

1. TWO-POINT CORRELATION FUNCTION AND THE OPE

Let us start with the simplest object – the two-point correlation function; the perturbative expansion for this object is known to a higher accuracy compared to more complicated correlators. Because of that, the formulation and application of the appropriate and reliable algorithms for the extraction of the hadron parameters from this correlator is becoming increasingly important.

The two-point function, i.e. the vacuum average of the T-product of two interpolating quark currents is the basic object for the sum-rule calculation of the decay constants of the heavy-light mesons such as B, B_s, D, D_s or their vector analogues. For instance, for heavy-light pseudoscalar currents $j_5=\overline{m}_b\bar{q}i\gamma_5 b$ (here $\overline{m}_b$ is the scale-dependent $\overline{\rm MS}$ mass of the heavy quark and M_b will denote its pole mass; the light-quark mass is neglected) one obtains

$$\Pi(p^2)=i\int d^4x\, e^{ipx}\left\langle\Omega\left|T\left(j_5(x)j_5^\dagger(0)\right)\right|\Omega\right\rangle \tag{1}$$

The Wilson OPE for the T-product and for the correlation function has the following form:

$$T\left(j_5(x)j_5^\dagger(0)\right)=C_0(x^2,\mu)\hat{1}+\sum_n C_n(x^2,\mu):\hat{O}(x=0,\mu): \tag{2}$$

and

$$\Pi(p^2)=\Pi_{\rm pert}(p^2,\mu)+\sum_n\frac{C_n}{(p^2-M_b^2)^n}\langle\Omega|:\hat{O}(x=0,\mu):|\Omega\rangle \tag{3}$$

Here the physical QCD vacuum $|\Omega\rangle$ is a complicated object which differs from perturbative QCD vacuum $|0\rangle$. The properties of the physical vacuum are characterized by the condensates – the nonzero expectation values of gauge-invariant operators over this physical vacuum:

$$\langle\Omega|:\hat{O}(0,\mu):|\Omega\rangle\neq 0. \tag{4}$$

The numerical estimates for the condensates may be found in [2, 3]. Here we only list the recent determinations of the lowest-dimension condensates which claim an extremely high accuracy:

$$\langle\Omega|\bar{q}q(2\ {\rm GeV}|\Omega\rangle^{\overline{\rm MS}}=(282\pm 2\ {\rm MeV})^3\ [9],\qquad \langle\Omega|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a,\mu\nu}|\Omega\rangle=0.013\pm 0.0016\,{\rm GeV}^4\ [10]. \tag{5}$$

The two-point function satisfies the dispersion representation (which requires subtractions not shown here)

$$\Pi(p^2)=\int\frac{ds}{s-p^2}\rho(s), \tag{6}$$

and may be calculated both using OPE (which gives it in the form $\Pi_{\rm OPE}(p^2)$) and using the sum over the hadron intermediate states (which gives it in the form $\Pi_{\rm hadr}(p^2)$). The sum rule is the statement that both forms represent the same quantity and thus should be equal to each other

$$\Pi_{\rm OPE}(p^2)=\Pi_{\rm hadr}(p^2). \tag{7}$$

The spectral densities for the two representations read

$$\rho_{\rm OPE}(s)=\left[\rho_{\rm pert}(s,\mu)+\sum_n C_n\delta^{(n)}(s-M_b^2)\langle\Omega|O_n(\mu)|\Omega\rangle\right],\qquad \rho_{\rm hadr}(s)=f_B^2M_B^4\delta(s-M_B^2)+\rho_{\rm cont}(s) \tag{8}$$

Here M_B denotes the heavy-meson mass, f_B is its decay constant defined as

$$\langle 0|j_5|B\rangle=f_BM_B^2. \tag{9}$$

The truncated OPE series has quark and gluon singularities and does not have the hadron ones; therefore, comparison of the truncated OPE and the hadron representation in (7) may be done in the region of p^2 far from hadron thresholds and resonances.

Performing the Borel transform which serves several purposes (suppressing the contribution of the excited states, killing the subtraction terms in the dispersion representation for $\Pi(p^2)$, improving the convergence of the perturbative expansion [1]) one arrives at the Borel image of the two-point function

$$\Pi_{\rm hadr}(\tau)=\int ds\exp(-s\tau)\rho_{\rm hadr}(s)=f_B^2M_B^4e^{-M_B^2\tau}+\int\limits_{s_{\rm phys}}^{\infty}ds\,e^{-s\tau}\rho_{\rm hadr}(s), \tag{10}$$

where $s_{\rm phys}=(M_{B^*}+M_P)^2$ is the physical continuum threshold, determined by the masses of hadrons which may appear as the intermediate states, and

$$\Pi_{\rm OPE}(\tau)=\int ds\exp(-s\tau)\rho_{\rm OPE}(s)=\int\limits_{m_b^2}^{\infty}ds\,e^{-s\tau}\rho_{\rm pert}(s,\mu)+\Pi_{\rm power}(\tau,\mu), \tag{11}$$

where power corrections $\Pi_{\rm power}(\tau,\mu)$ are given via the condensates and radiative corrections to them.

The sum rule now takes the form

$$\Pi_{\rm OPE}(\tau)=\Pi_{\rm hadr}(\tau). \tag{12}$$

Recall that the hadron (i.e. full-QCD) representation $\Pi_{\rm hadr}(\tau)$ is an infinite sum of the exponential terms, whereas power corrections in $\Pi_{\rm OPE}(\tau)$ contain polynomials in τ multiplied by $\exp(-M_b^2\tau)$. Therefore the truncated OPE provides a good description of $\Pi_{\rm hadr}(\tau)$ at "not too large" values of τ. This determines the choice of the *Borel window* – the working τ-range where the OPE gives an accurate description of the exact correlator (i.e., all higher-order radiative and power corrections are under control) and at the same time the ground state gives a "sizable" contribution to the correlator.

The best-known 3-loop calculations of the perturbative spectral density [11] have been performed in form of an expansion in terms of the $\overline{\rm MS}$ strong coupling $\alpha_s(\mu)$ and the pole mass M_b:

$$\rho_{\rm pert}(s,\mu)=\rho^{(0)}(s,M_b^2)+\frac{\alpha_s(\mu)}{\pi}\rho^{(1)}(s,M_b^2)+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\rho^{(2)}(s,M_b^2,\mu)+\cdots. \tag{13}$$

An alternative option [12] is to reorganize the perturbative expansion in terms of the running $\overline{\rm MS}$ mass, $\overline{m}_b(\nu)$, by substituting M_b in the spectral densities $\rho^{(i)}(s,M_b^2)$ via its perturbative expansion in terms of the running mass $\overline{m}_b(\nu)$

$$M_b=\overline{m}_b(\nu)\left(1+\frac{\alpha_s(\nu)}{\pi}r_1+\left(\frac{\alpha_s(\nu)}{\pi}\right)^2r_2+\dots\right). \tag{14}$$

As noticed in [12, 13], two different scales, μ and ν, naturally emerge when reorganizing the perturbative expansion from the pole b-quark mass to the running b-quark mass. In our discussion we do not distinguish between these scales, but in practical calculations the scales have been treated independently.

Advanced algorithms for an isolation of the ground-state contribution

The hadron representation contains the sum over all hadron intermediate states, whereas we are primarily interested in the ground state contribution. To exclude the excited-state contributions, one adopts the *duality Ansatz*: all contributions of excited states are counterbalanced by the perturbative contribution above an *effective continuum threshold,* $s_{\rm eff}(\tau,\mu)$ which differs from the physical continuum threshold. Applying the duality assumption yields:

$$f_B^2 M_B^4 e^{-M_B^2\tau} = \int\limits_{m_b^2}^{s_{\rm eff}(\tau,\mu)} ds\, e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu) \equiv \Pi_{\rm dual}(\tau, s_{\rm eff}(\tau,\mu)). \qquad (15)$$

The rhs is the *dual correlator* $\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau))$ (we shall not explicitly write μ as an argument of $s_{\rm eff}$ but this dependence should be kept in mind). *Obviously, even if the QCD inputs $\rho_{\rm pert}(s,\mu)$ and $\Pi_{\rm power}(\tau,\mu)$ are known, the extraction of the decay constant requires $s_{\rm eff}(\tau,\mu)$.* Let us emphasize, that the effective threshold should be the function of τ and μ: (i) one can easily check that $s_{\rm eff}$ should depend on τ in order the τ-dependences of the r.h.s. and the l.h.s. of (15) match each other; (ii) since the truncated OPE is used in the r.h.s. of (15), the effective threshold also depends on the choice of the scale μ.

In early applications of the method of sum rules, it was common to use the approximation $s_{\rm eff}(\tau) = const$; the value of this constant has been fixed by requiring the maximal stability (i.e. the least unphysical dependence of the hadron observable on the Borel parameter τ). This procedure proved to work reasonably well, although it did not allow one to probe the uncertainty of the extracted hadron parameter induced by using the approximation of a constant effective continuum threshold.

It should be emphasized that even if the OPE for the correlation function is known with very high accuracy in the Borel window, the hadron parameters can still be determined with some uncertainty which reflects the limited intrinsic accuracy of the method of sum rules. We refer to the corresponding uncertainty as to the *systematic uncertainty*. The latter is related to the adopted prescription for fixing the effective continuum threshold $s_{\rm eff}(\tau)$.

As the accuracy of the OPE for the correlation functions has increased, one faced the acute necessity to provide more accurate and reliable procedures for the extraction of hadron parameters: gaining control over the systematic uncertainties has become mandatory [8].

The results of [14] demonstrated that in those cases where the bound-state mass M_B is known, one can use it and improve the accuracy of the decay constant. We introduce the *dual invariant mass* $M_{\rm dual}$ and the *dual decay constant* $f_{\rm dual}$

$$M^2_{\rm dual}(\tau) \equiv -\frac{d}{d\tau}\log \Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)), \qquad f^2_{\rm dual}(\tau) \equiv M_B^{-4} e^{M_B^2\tau}\,\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)). \qquad (16)$$

The deviation of $M_{\rm dual}(\tau)$ from M_B measures the contamination of the dual correlator by excited states.

Starting with any trial function for $s_{\rm eff}(\tau)$ and minimizing the deviation of $M_{\rm dual}$ from M_B in the τ-window yields a variational solution for $s_{\rm eff}(\tau)$. As soon as the latter is found, one readily obtains the corresponding decay constant f_B from (15).

We consider polynomials in τ and obtain their parameters by minimizing the squared difference between $M^2_{\rm dual}$ and M_B^2 in the τ-window:

$$\chi^2 \equiv \frac{1}{N}\sum_{i=1}^{N}\left[M^2_{\rm dual}(\tau_i) - M_B^2\right]^2. \qquad (17)$$

As shown in several exactly solvable models, the band of the estiamtes for f_B corresponding to the variational solutions for linear, quadratic, and cubic trial $s_{\rm eff}(\tau)$, provides a realistic estimate for the systematic uncertainty of the decay constant [15, 16].

The resulting f_B obtained according to the procedure described above is sensitive to the input values of all the OPE parameters (quark masses, α_s, the condensates) which are known with some uncertainties thus yielding the *OPE-related uncertainty* of f_B. To obtain the latter, one assumes the Gaussian distributions for the OPE parameters mentioned above. Moreover, because of the truncation of the OPE series, the decay constants exhibit an unphysical dependence on the precise value of the renormalization scales μ. A priori, any choice of the scale is equivalently good; therefore, we average over the scale in some intervals assuming the *uniform* distribution of μ.

• Another simple algorithm for fixing the τ-dependent effective threshold in the Borel sum rule has been recently adopted in [17]: for each value of τ the authors calculated $M_{\rm dual}(\tau)$ *neglecting* the τ-dependence of $s_{\rm eff}(\tau)$ and then easily obtain $s_{\rm eff}$ by solving the equation $M_{\rm dual}(\tau) = M_B$. Obviously, the resulting effective thresholds do depend on τ; neglecting their τ-dependence while calculating the dual mass leads to some intrinsic inconsistencies. Following our old idea, we tested the algorithm of [17] in a quantum-mechanical potential model for the case of a potential which contains the confining and the Coulomb parts [15]. This analysis shows that in quantum mechanics the algorithm with the variational solutions described above provides more reliable and accurate estimates for the decay constants of the heavy-light mesons compared with the algorithm of [17].
• An interesting approach to the extraction of the ground-state parameters within the finite-energy sum rule has been formulated and applied to the decay constants of heavy-light and heavy-heavy mesons in [18]. We have also tested this algorithm in the potential model [15]. For the potential-model parameters appropriate for for heavy-light mesons the algorithm of [18] was shown to provide rather accurate estimates for the decay constants such that the "invisible" systematic error remains at a few percent level only.

Charm sector

For the extraction of the decay constants of the charmed pseudoscalar and vector mesons, one makes use of the best-known three-loop expression for the spectral densities of the two-point functions for pseudoscalar and vector currents. The OPE in terms of the pole mass M_b calculated in [11] does not exhibit a perturbative hierarchy, therefore one rearrange the OPE in terms of the running $\overline{\rm MS}$-mass [12]. Then, the perturbative hierarchy of the correlation function starts to depend on μ; this feature allows one to choose the range of μ where the perturbative hierarchy is visible. The negative effect of this rearrangement of the perturbative expansion is that, because of the truncation of the OPE series, the extracted decay constants acquire an unphysical dependence on the scale μ. In the charm sector this however does not lead to any serious problems. Figure 1 shows the dependence of the decay constants of the charmed pseudoscalar and vector mesons for the central values of all other OPE parameters after applying the algorithm for fixing the effective thresholds described above. One can see a weak μ-dependence of the decay constants of the pseudoscalar mesons mesons, whereas for vector mesons this μ-dependence is more pronounced. Averaging over the OPE parameters in their respective intervals and over the scale in the range $1 \le \mu[GeV] \le 3$ one arrives at the following results [19]

$$\begin{aligned} f_D &= (208.3 \pm 7.3_{\rm OPE} \pm 5_{\rm syst})\ {\rm MeV}, \qquad & f_{D_s} &= (246.0 \pm 15.7_{\rm OPE} \pm 5_{\rm syst})\ {\rm MeV} \\ f_{D^*} &= (252.2 \pm 22.3_{\rm OPE} \pm 4_{\rm syst})\ {\rm MeV}, \qquad & f_{D_s^*} &= (305.5 \pm 26.8_{\rm OPE} \pm 5_{\rm syst})\ {\rm MeV}. \end{aligned} \tag{18}$$

For the ratio we reported $f_{D^*}/f_D = 1.221 \pm 0.080_{\rm OPE} \pm 0.008_{\rm syst}$, which compares nicely with the lattice QCD result $f_{D^*}/f_D = 1.20 \pm 0.02$. The results for the charmed mesons from other sum-rule analyses [17, 20] agree well with each other and with the results from lattice QCD [21].

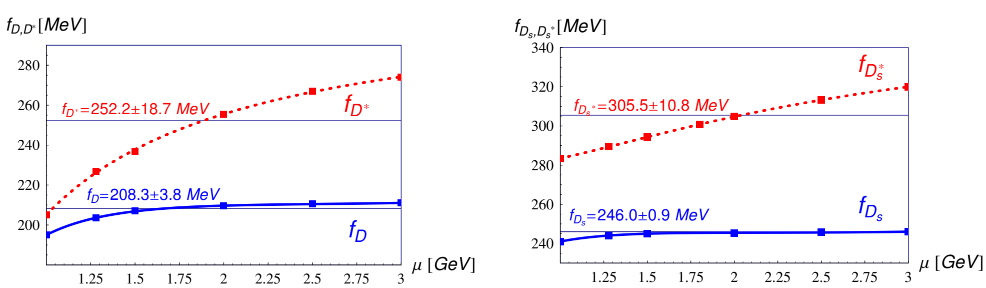

FIGURE 1. Decay constants of D, D_s D^* and D_s^* mesons depending on the scale μ.

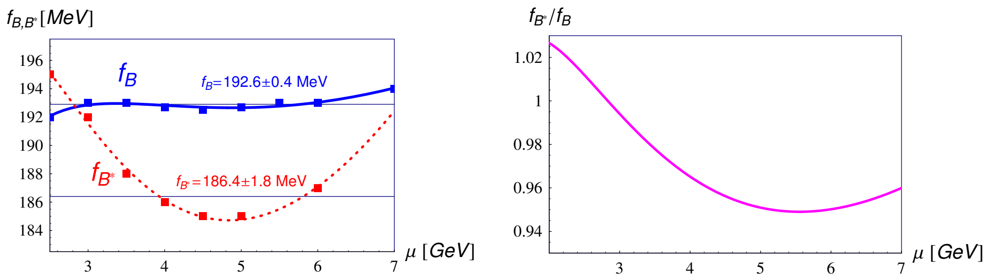

FIGURE 2. Decay constants of B and B^* mesons and the ratio f_{B^*}/f_B depending on the scale μ.

Beauty sector

Similar to the charm sector, the OPE for pseudoscalar and vector currents containing the b-quark, does not show any perturbative hierarchy; there is no reason to assume that the unknown higher-order perturbative contributions are small. Rearranging the perturbative expansion in terms of the running mass introduces the dependence of the scale μ and opens the possibility to choose the working range of μ in which the perturbative hierarchy is explicit thus allowing to hope the unknown higher orders do not contribute substantially to the correlation function.

In the b-sector one encounters two interesting features of the sum-rule analysis:

• The sum-rule results for the beauty-meson decay constants correlate very strongly with the b-quark mass [22]

$$\delta f_B/f_B \approx -8\,\delta m_b/m_b, \tag{19}$$

$m_b \equiv \overline{m}_b(\overline{m}_b)$. Making use of $m_b = 4.18 \pm 0.03$ GeV [23] leads to $f_B > 210$ MeV, in clear tention with the recent lattice QCD results for $f_B \sim 190$ MeV. Combining our sum-rule analysis with f_B and f_{B_s} from lattice QCD yields [22]

$$m_b = (4.247 \pm 0.027 \pm 0.011_{(\mathrm{syst})})\ \mathrm{GeV}. \tag{20}$$

The sum-rule results for the decay constants corresponding to this value of the b-quark mass read

$$f_B = (192.0 \pm 14.3_{\mathrm{OPE}} \pm 3.0_{\mathrm{syst}})\ \mathrm{MeV}, \qquad f_{B_s} = (228.0 \pm 19.4_{\mathrm{OPE}} \pm 4.0_{\mathrm{syst}})\ \mathrm{MeV} \tag{21}$$

• For the decay constant of B^*, one observes an unexpectedly strong μ-dependence [24]: Averaging over the scale range $3 < \mu[\mathrm{GeV}] < 6$ leads to

$$f_{B^*}/f_B = 0.923 \pm 0.059, \qquad f_{B_s^*}/f_{B_s} = 0.932 \pm 0.047.$$

Taking into account only low-scale results for $2.5 < \mu[\mathrm{GeV}] < 3.5$, yields $f_{B^*}/f_B = 0.994 \pm 0.01$. The sum-rule analysis [17] also gives indications that $f_{B^*}/f_B \leq 1$ (ses Table II of [17]). Surprisingly, the QCD sum-rule prediction for f_{B^*}/f_B is below the corresponding results from lattice QCD, which seem to favour a value slightly above unity [21, 25]. Clearly, such tension calls for further detailed investigations.

μ-dependence of the physical quantities

The heavy-light correltors are known with an impressive three-loop accuracy and are therefore rather weakly sensitive to the variations of the scale. Nevertheless, the *dual correlator* of the vector currents which includes the low-energy region of the Feynman diagrams only and, respectively, the vector-meson decay constants are rather sensitive to the choice of the scale. In many cases this scale-dependence is the main sourse of the OPE uncertainty in the decay constants. We should mention that in some publications the μ-dependence is treated in a specific way [20]: one just chooses one scale at which the decay constant has, e.g., an extremum in μ, and provides the results for this very scale assigning no theoretical uncertainty to the scale fixing. This of course reduces strongly the total uncertainty of the decay constant obtained with the sum-rule technique but from our point of view such a treatment is not justified: the (unphysical) μ-dependence is an effect of the truncation of the OPE series and thus reflects an essential feature of QCD. Any of the scale for which a reasonable perturbative hierarchy is seen, may be used for the determination of the hadron parameter; the unpleasant μ-dependence of the sum-rule results should be thus properly reflected in the theoretical uncertainty of the hadron parameter obtained using a QCD sum rule.

SUM RULES FOR THREE-POINT VACUUM CORRELATION FUNCTIONS

Let us now discuss the calculation of the meson elastic and transition form factors from the three-point vacuum correlation functions [26, 27]. The basic object in this case has the form

$$\Gamma(p^2,p'^2,q^2)=\int\langle\Omega|T(j(x)j(0)j(y))|\Omega\rangle\exp(-ipx)\exp(-ip'y)dxdy. \tag{22}$$

The three-point Green function in full QCD contains the double pole related to the mesons in the p^2 and p'^2-channels in the timelike region. The residue in this double pole is the form factor of interest. The Green function in the spacelike region may be calculated using the same method as the two-point function, i.e. by performing the OPE. One represents the Green function $\Gamma(p^2,p'^2,q^2)$ as a double spectral integral in p^2 and p'^2, performs the double Borel transform $p^2\to\tau$ and $p'^2\to\tau'$ (which, similar to the two-point function, kills the subtraction terms and suppresses the contributions of the excited states), equate to each other the OPE and the hadron representations for $\Gamma(p^2,p'^2,q^2)$, and use duality property to isolate the ground-state contribution, thus relating the meson form factor to the low-energy region of the triangle diagrams of perturbative QCD and power corrections given through the condensates. For instance, the pion elastic form factor, in which case one sets $\tau=\tau'$, has the form [27]

$$\begin{aligned}F_\pi(Q^2)\,f_\pi^2&=\int\limits_0^{s_{\rm eff}(Q^2,\tau)}\int\limits_0^{s_{\rm eff}(Q^2,\tau)}{\rm d}s_1\,{\rm d}s_2\,\Delta_{\rm pert}(s_1,s_2,Q^2)e^{-\frac{s_1+s_2}{2}\tau}+\frac{\langle\alpha_s G^2\rangle}{24\pi}\,\tau+\frac{4\pi\alpha_s\langle\bar{q}q\rangle^2}{81}\,\tau^2\left(13+Q^2\tau\right)+\cdots,\\ \Delta_{\rm pert}(s_1,s_2,Q^2)&=\Delta^{(0)}(s_1,s_2,Q^2)+\alpha_s\Delta^{(1)}(s_1,s_2,Q^2)+\cdots.\end{aligned} \tag{23}$$

An essential feature of the three-point sum rule is that the effective threshold now depends on the Borel parameter τ and the momentum transfer Q [28–30]; obviously, one faces a serious problem of finding appropriate algorithms for fixing $s_{\rm eff}(Q^2,\tau)$. It should be understood that the effective continuum threshold for the form factor differs from the effective threshold for the decay constant.[1]

For large Q^2, power corrections calculated in terms of the local condensates rise as polynomials with Q^2, thus preventing a direct use of the sum rule (23) at large Q^2. There are essentially only two possibilities to study the region of large Q^2 starting with the vacuum correlators:

- use nonlocal condensates which are aimed at the resummation of the local condensate effects [31, 32].
- work in the so-called local-duality (LD) limit $\tau=0$ [31]. A specific feature of this limit is that all power corrections vanish in this limit and details of non-perturbative dynamics are hidden in one complicated object – the Q^2-dependent effective threshold $s_{\rm eff}(Q^2)$.

A similar treatment may be performed for, e.g., the $\pi^0\to\gamma\gamma^*$ transition form factor [33–35] for which one obtains the single spectral representation in the LD limit:

$$F_{\pi\gamma}(Q^2)\,f_\pi=\int\limits_0^{\bar{s}_{\rm eff}(Q^2)}{\rm d}s\,\sigma_{\rm pert}(s,Q^2) \tag{24}$$

Due to properties of the spectral functions $\Delta_{\rm pert}(s_1,s_2,Q^2)$ and $\sigma_{\rm pert}(s,Q^2)$, the form factors obey the factorization theorems

$$F_\pi(Q^2)\to 8\pi\alpha_s(Q^2)f_\pi^2/Q^2,\qquad F_{\pi\gamma}(Q^2)\to\sqrt{2}f_\pi/Q^2,\qquad f_\pi=130\ {\rm MeV} \tag{25}$$

as soon as the effective thresholds satisfy

$$s_{\rm eff}(Q^2\to\infty)=\bar{s}_{\rm eff}(Q^2\to\infty)=4\pi^2 f_\pi^2. \tag{26}$$

Remarkably, due to the QCD factorization theorems for the hard form factors, the effective thresholds at $Q^2\to\infty$ are given through the decay constants of the participating mesons. It should be emphasized that the only feature of theory relevant for this property of $s_{\rm eff}(Q^2)$ is *factorization* of hard form factors.

[1] The effective thresholds for the baryon form factors are strongly sensitive to the choice of the interpolating current for a specific baryon.

For finite Q^2, the effective thresholds $s_{\rm eff}(Q^2)$ and $\bar{s}_{\rm eff}(Q^2)$ depend on Q^2 and differ from each other [36, 37]. Nevertheless, setting $s_{rmeff}(Q^2) = s_{rmeff}(Q^2 \to \infty)$ for all not too small Q^2 [27] provides an approximate *parameter-free* prediction for the form factors which is becoming increasingly accurate as soon as Q^2 increases. The results of [37] give convincing evidences that $s_{\rm eff}(Q^2)$ and $\bar{s}_{\rm eff}(Q^2)$ are close to their asymptotic values already at relatively low values $Q^2 \approx 4-8$ GeV^2.

Thus, the LD approximation for the form factors—which requires as its crucial ingredient the knowledge of $O(1)$ and $O(\alpha_s)$ double spectral densities—is increasingly accurate in the region not too close to zero recoil. The LD approximation is very promising for the application to, e.g., heavy-to-light weak form factors. A still missing ingredient here is the two-loop $O(\alpha_s)$ double spectral density of the triangle diagram for different currents and arbitrary quark masses in the loop. This is a really challenging calculation which however opens the possibilities of very interesting applications. So far the only known results correspond to all massless quarks in the loop [38] and to HQET [39, 40].

SUM RULES FOR THE EXOTIC POLYQUARK CURRENTS

The OPE for the correlation functions of the exotic polyquark currents involving 4 (or 5) quark fields of the type

$$D(x) = \bar{q}_1(x)\hat{O}q_2(x)\bar{q}_3(x)\hat{O}q_4(x) \tag{27}$$

where $\hat{O}$ is an appropriate combination of the Dirac matrices and possibly also of the (covariant) derivatives, have specific features compared to the OPE for the bilinear currents of the form $j(x) = \bar{q}_1(x)\hat{O}q_2(x)$ used for usual "nonexotic" mesons. Namely, the lowest-order $O(1)$ contribution to the OPE for any correlator involving the exotic current, e.g. $\Pi_{DD} = \langle 0|T(D(x)D(0)|0\rangle$, is given by the disconnected diagrams. As known from the general features of the Bethe-Salpeter equation and also emphasized recently by Weinberg [41], these disconnected diagrams are not related to the exotic bound states. The *connected* diagrams relevant for the exotic states emerge in the OPE for any correlator at the order $O(\alpha_s)$ and higher; therefore for the analysis of the exotic states the knowledge of the radiative corrections is mandatory. This makes the analysis of the exotic states a more technically involved problem than the analysis of the normal hadrons.

Nevertheless, due to the fact that the observed exotic states are narrow, the procedure of extracting their parameters from the OPE has the same features and the same challenges as for the normal hadrons. Our experience in the analysis of the usual hadrons proves that a truncated OPE for the correlation function does not allow one to study at the same time both the *existence* of the isolated ground state and of its *properties*. However, if the mass of the narrow bound state is known, the method of sum rules allows one to obtain reliable predictions for its decay constants and the form factors.

Structure of the exotic tetraquark states

Obviously, the exotic tetraquark states may have a rather complicated "internal" structure; two most popular scenarios of this structure are a confined tetraquark state (i.e. a bound state in a confining potential between the two color-triplet diquarks) and a molecular "nuclear-physics like" bound state in the system of two colorless mesons.

However, an important question about the structure of the exotic state—which to large extent determines also its production mechanism—is not easy to answer [42]: (i) by a combined color-spinor Fierz rearrangement of the tetraquark interpolating current $D(x)$ one can write it either in diquark-antidiquark or meson-meson form; (ii) the same quantum numbers of the exotic interpolating current may be obtained by different combinations of its diquark-antidiquark or meson-meson bilinear parts.

The simplest characteristic of a usual meson is its decay constant, i.e. the transition amplitude between the vacuum and the meson induced by its interpolating current; for a heavy quarkonium state the decay constant is analogous to its wave function at the origin $\psi(r=0)$.

For an exotic tetraquark state one should considers the *connected* self-energy functions

$$\Pi_{DD} = \langle 0|T(D(x)D(0)|0\rangle \equiv \langle DD\rangle \tag{28}$$

and study the corresponding sum rules. However, for an exotic state one may obtain a set of the decay constants, related to different structure of the interpolating current with the quantum numbers of the exotic tetraquark of interest.

The answer to the question of the dominant structure of the tetraquark may be given only by the analysis of a large set of the decay constants.

• As the first step, one needs to study systematically the interpolating currents for tetraquark currents with different quantum nembers. As the next step one can calculate the set of Π_{DD}. Because of the factorization property of the two-point function of the local tetraquark currents [43], the radiative corrections to Π_{DD} are given via radiative corrections to the various two-point functions of the bilinear quark currents. For some of these two-point functions (namely, $\langle VV\rangle$ and $\langle AA\rangle$) the radiative corrections are well-known, for some of them (such as $\langle TT\rangle$, T is the tensor bilinear current) these corrections should be calculated.

• Then, the set of the sum rules for different two-point functions Π_{DD} should be studied and only then the answer about the structure of the observed narrow exotic candidates may be obtained. Especially interesting cases here are the narrow charged tetraquark $Z^-(4430)$ ($J^P = 1^+$ and the width $\simeq 45$ MeV, valence-quark content $\bar{c}c\bar{u}d$) and X(3872) ($J^{PC} = 1^{++}$ X(3872), the width < 24 MeV).

Another interesting possibility—so far not discussed in the literature—is considering *nonlocal* interpolating currents for the exotic mesons. The nonlocality of the interpolating currents should allow one to access in a better way subtle details of the tetraquark structure.

Strong fall-apart decays of the exotic tetraquark states

In the last decade, QCD sum rules have been extensively applied to the analysis of strong decays of exotic multiquark states (see e.g. [44, 45] and references therein). The basic object for the analysis of these decays in QCD is the three-point functions of the type

$$\Gamma(p,p',q) = \int \langle 0|T(D(0)j(x_1)j(x_2)|0\rangle \exp(-ip'x_1 - iqx_2)dx_1dx_2. \tag{29}$$

This correlator contains the triple-pole in the Minkowski region

$$\Gamma_{\rm hadr}(p,p',q) = \frac{f_X f_{M_1} f_{M_2} g_{XM_1M_2}}{(p^2-M_X^2)(p'^2-M_1^2)(q^2-M_2^2)} + \cdots \tag{30}$$

where dots stay for less singular terms. Here $g_{XM_1M_2}$ is the three-hadron coupling which describes the $X \to M_1M_2$ transition; f_X, f_{M_1}, and f_{M_2} are the decay constants of the mesons describing the strength of their their interaction with the interpolating current $\langle X|D(0)|0\rangle = f_X$ and $\langle M_{1,2}|j_{1,2}(0)|0\rangle = f_{1,2}$ (we omit here all Lorentz indices and for simplicity neglect the spins of the hadrons and of the interpolating currents). The OPE allows one to calculate the expansion of this correlator at the spacelike momenta far from the hadron thresholds. Again, the leading contribution in α_s is given by a disconnected diagram (see Fig.3a) which factorizes and does not depend on the momentum of the exotic current p^2 at all:

$$\Gamma_{\rm OPE}(p^2,p'^2,q^2) = \Pi(p'^2)\Pi(q^2) + \alpha_s\Gamma_{\rm connected}(p^2,p'^2,q^2) \tag{31}$$

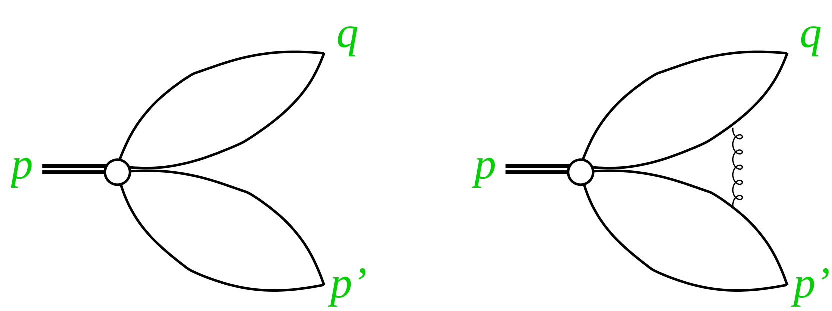

FIGURE 3. (a) The disconnected $O(1)$ diagram which does not depend on the variable p^2 relevant for the tetraquark properties; (b) One of the lowest-order connected $O(\alpha_s)$ diagram which contributes to the tetraquark decay amplitude.

Performing the Borel transform $p^2 \to \tau$, which comprises one of the steps of the sum-rule analysis, we see that the Borel image of the disconnected leading-order contribution vanishes.[2] Therefore any attempt to extract the tetraquark decay amplitude from the leading-order contribution is inconsistent. Relevant for the exotic-state properties are the $O(\alpha_s)$ corrections which are technically very difficult. This is a difficult calculation but it should be done before one may hope to get reliable predictions for the tetraquark properties. So far these corrections have been calculated only for the three-point function of the bilinear currents in two cases (i) for massless quarks and (ii) for infinitely heavy active quark and a massless spectator. For the $O(\alpha_s)$ corrections to the three-point functions Γ, involving one tetraquark and two bilinear currents, no results exist in the literature.

Nevertheless, the common feature of *all* previous calculations of these decays within QCD sum rules (e.g. [45, 46]) was the attempt to study the tetraquark (and pentaquark) decays basing on the factorizable leading-order contribution which intrinsically has no relationship with the tetraquark properties (which is clear both from the factorization property $\Gamma(p,p',q) = \Pi(p'^2)\Pi(q^2)$ and from the large-N_c behaviour of the QCD diagrams emphasized by Weinberg [41]. Therefore the existing analyses should be strongly revised by calculating and taking into account the nonfactorizable two-loop $O(\alpha_s)$ corrections.

Nonzero results based on the leading-order correlation function may be obtained only by a trick. Let us consider e.g. the decay $Z \to \psi' + \pi^-$. One makes use of the tetraquark current $j(x) = \bar{c}(x)c(x)\bar{u}(x)d(x)$ (we again omit the Dirac matrices for simplicity). The corresponding three-point correlation function of interest is

$$\Gamma(p^2,p'^2,q^2) \quad = \quad \int d^4x d^4y \exp(-ip'x)\exp(-iqy)\langle 0|T(\bar{c}(0)c(0)\bar{u}(0)d(0), \bar{c}(x)c(x), \bar{d}(y)u(y))|0\rangle. \qquad (32)$$

A nonzero result for the Borel transform of the disconnected zero-order contribution may be obtained by first considering the soft-pion limit $q \to 0$, i.e. $p' = p$, which gives for the disconnected contribution $\Pi(p^2)\Pi(0)$ and then performing the Borel transform $p^2 \to \tau$. However, the decay rate obtained in this way is not really trustworthy.

We therefore conclude that *the "fall-apart" decay mechanism of exotic hadrons differs from the decay mechanism of the ordinary hadrons and requires the appropriate treatment within QCD sum rules. The calculation of the radiative corrections is mandatory for a reliable analysis of the properties of the exotic states.*

SUMMARY AND OUTLOOK

In the recent years, great progress has been seen both in the calculations of the OPE series for various correlation functions and in the direction of formulating advanced algorithms for the extraction of the individual hadron parameters from these correlators. We could not discuss all the developments in this talk but let us try to mention in this summary the interesting open issues to be addressed in the future analyses:

• Let us recall that combining moment QCD sum rules with experimental/lattice data gives the most accurate estimates of the heavy-quark masses [47].

• *Hadron properties from 2-point functions:*

a. We have seen a visible progress in developing the new algorithms for extracting ground state parameters from the OPE of the correlators and gaining control over the systematic errors of the decay constants (finite-energy sum rules, Borel sum rules). Although it seems impossible to predict both masses and decay constants with a controlled accuracy, using the mass of the ground state as input, systematics can be controlled).
b. We have encountered interesting puzzles in the b-sector:
(i) The b-quark mass 4.18 GeV [23] when used in the Borel sum rules for f_B leads to tension with lattice results for f_B.
(ii) Unexpectedly strong scale-dependence of decay constants of vector mesons and of f_{B^*}/f_B even using the $O(\alpha_s^2)$ correlation function.
c. Calculation of the decay constants of heavy-quarkonium states within the method of QCD sum rules is still not fully settled: The problem here is that the OPE for the doubly-heavy correlation functions contain *relatively small*

[2] We emphasize that for the decay of a usual hadron, the $O(1)$ contribution is given by a triangle diagram which depends on all three variables p^2, p'^2, q^2 and therefore of course does not vanish under the Borel transform $p^2 \to \tau$; for the decays of the usual hadrons the $O(1)$ contribution of the perturbative QCD indeed provides the dominant contribution to the decay of interest.

nonperturbative power corrections. Therefore in QCD, the structure of OPE for the heavy-quarkonium system is somewhat similar to the structure of OPE for a purely coulomb system. Obviously, the algorithms adopted and tested for light or heavy-light hadrons in which cases the nonperturbative corrections are large, may work differently for heavy quarkonium states. This feature may be the origin of the tensions between the sum-rule predictions and the results from lattice QCD and other nonperturbative approaches for e.g. the decay constants of B_c mesons and some charmonium states [20, 48]. A more critical analysis of the procedures of an isolation of the ground-state contribution from the correlation function and in particular of the way of obtaining the systematic uncertainties is necessary.

d. Since the accuracy of the isolation of the ground-state contribution from the correlation function can be controlled, one may try to apply the method for the analysis of the excited states. Very little efforts in this direction have been done so far.

• *Meson elastic and transition form factors from three-point functions*

The Borel sum rules at $\tau = 0$ (the so-called local duality limit) open an interesting possibility of obtaining parameter-free predictions for the elastic and the transition form factors of light mesons in a broad range of the momentum transfers. The crucial ingredients necessary for these calculations are the $O(1)$ and $O(\alpha_s)$ spectral densities of the triangle diagrams. As soon as these are known, the effective thresholds are determined in a unique way by the QCD factorization theorems for hard form factors. Assuming the effective thresholds to weakly depend on the momentum transfers—a hypothesis which finds support in the data for the pion form factors—one obtains the parameter-free predictions for the form factors in a broad range of the momentum transfers. Our analysis suggests that these representations for the form factors work with a few percent accuracy for $Q^2 \geq$ a few GeV^2. It seems very promising to apply the same ideas to heavy-to-light transition form factors. The main problem here is the necessity to calculate the radiative corrections to the triangle diagrams which is a very difficult task which needs serious efforts. As soon as this ambitious task is fulfilled, QCD sum rules could provide parameter-free predictions for the form factors, increasingly accurate with increase of Q^2.

• *Baryon elastic and transition form factors*

The calculations for baryons are obviously technically extremely involved. Many sum-rule analyses of the baryon elastic and transition form factors have been presented in the recent years (see e.g. [49–52] and references therein). With a few exceptions (e.g. [49]), these calculations are based on the leading-order $O(1)$ correlation functions and use the traditional approaches to fix the effective thresholds, usually neglecting the τ- and Q^2-dependence of the latter. These analyses are expected to provide reasonable ball-park estimates for the form factors; however, in most of the cases, the estimates of the OPE-errors (related to the uncertainties of the QCD parameters, to the missing radiative corrections, and in particular to a strong dependence on the scale μ) and the systematic errors, related to the adopted procedures of fixing the effective continuum thresholds) are not done properly. Many efforts are still to be done in the domain of the baryon form factors.

• *Three-meson strong couplings of the type $g_{D^*D\pi}$*

These quantities have been extensively addressed using three-point vacuum correlators and the corresponding sum rules. Again, the radiative corrections to the correlation functions have not been taken into account. Moreover, the results for the decay constants require extrapolations over large ranges of the momentum transfers. Therefore, one cannot expect good accuracy of these estimates. In many cases, the results from sum rules lead to an unrealistic picture of the $SU(3)$-breaking effects (see [53] and refs therein). For a real progress, one needs the calculation and the inclusion of the radiative corrections to the appropriate three-point functions.

• *Properties of the exotic tetraquark states*

The “fall-apart” decay mechanism of exotic hadrons differs from the decay mechanism of the ordinary hadrons and requires the appropriate treatment within QCD sum rules. In distinction to the decays of the usual hadrons, where the knowledge of the radiative corrections is necessary for improving the accuracy of the sum-rule form factor calculations, the calculation of the radiative corrections is *mandatory* for a reliable analysis of the properties of the exotic states. The decays of the exotic states are intrinsically unrelated to the $O(1)$ disconnected correlation functions; the results obtained from these O(1) correlators cannot be treated as fundamental and reliable.

From this summary of the recent advances and still open issues it seems obvious that the future progress in the sum-rule calculations of the properties of the usual and the exotic hadrons will be related (i) to the calculations of the radiative corrections to the correlation functions and (ii) to further development of the appropriate algorithms for the extraction of the properties of the individual hadrons from these correlators.

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